Symmetrical Measuring: An Approach to Teaching Elementary School Mathematics Informed by Yup'ik Elders

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What would the curriculum look like if it were developed from the perspective of measuring? Without formal tools, the Yup'ik Eskimos of Alaska used their body as a measuring device and employed ratios extensively in their daily practices. *Math in a Cultural Context* is developing curriculum materials based on Yup'ik Elders use of mathematics. This paper describes a hypothesised learning/teaching sequence that is grounded in real life experience and linked to the mathematics in the classroom. Activities that were trialled in classrooms at a K-12 school in interior Alaska are also reported.

The achievement gap between Alaska Native (AN) students and their mainstream counterparts is of growing concern, especially in the area of mathematics. The National Assessment of Education Progress (NAEP) data showed that on average, performance of AN grade 4 and grade 8 students' performance was considerably lower than white Alaskans. Although the gap exists across all content strands, performance was weakest in the areas of measurement, number properties and operations (National Center for Education Statistics, 2010). State based assessment (SBA) data further highlights the disparity in achievement especially in rural Native school districts. Over 40% of students in many of these school districts (e.g., Yupiit, and Alaska Gateway) do not meet State proficiency standards in mathematics (Alaska Department of Education and Early Development, n.d.).

Factors associated with low performance in many rural districts include high rates of poverty, limited English proficiency, high teacher turnover, and cultural discontinuity. Some school districts have adopted mainstream curricula and in some instances, adopted international texts (e.g., Singapore Math) in an attempt to boost student performance and achievement (Juneau School District, n.d.). The reliance on textbooks has resulted in real life examples of mathematics which lack cultural relevance for AN students. This in turn creates obstacles for their learning and understanding of mathematics, as they are unable to see the usefulness of mathematics in their daily lives.

Math in a Cultural Context[1] (MCC) is a set of federally funded projects which aims to improve the mathematics performance of Alaskan Native students. Central to MCC is its long-term collaboration with Yup'ik Elders, teachers and Alaskan school districts. The wisdom of Yup'ik Elders and their everyday practice, which includes feats of ocean and star navigation without Western instrumentation, highlights their accomplishments in practical intelligence using mathematics. Yet their children and grandchildren struggle with school mathematics.

[1] More information can be found on the project website: http://www.uaf.edu/mcc/about/

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 661–668. Sydney: MERGA.

The National Mathematics Advisory Panel (2008) calls for students to learn topics in a coherent and effective manner. Hence, there is a need to bridge elders' knowledge to the teaching of school mathematics. Indeed, MCC includes contextual knowledge to make the "math" more familiar and relevant outside of the classroom (Lipka, Yanez, Andrew-Ihrke, & Adam, 2009). Experimental studies (e.g., Kisker et al., 2012; Lipka & Adams, 2004) on culturally relevant mathematics curriculum has shown statistically significant gains in mathematics for rural and urban, both Alaska Native and other students.

Mathematics from a Measurement Perspective

A Western view of learning mathematics commences with whole number followed by fractions, while measurement is a separate content strand (Alaska Board of Education and Early Development [ABEED], 2012). Measurement, researchers contend, can provide a conceptual base from which to develop understanding of number and geometry (Barrett et al., 2012), and fractions, ratios and early algebraic thinking (Davydov, 1991) through quantitative reasoning.

Davydov (1991) suggests that concepts such as fractions are made inaccessible when symbolic mathematics (e.g., $\frac{m}{n}$) is separated from concrete association with the process of measuring. The concept of fractions as it is introduced in the classroom through the division of concrete objects into equal parts, is becoming increasingly separated from their fundamental origins of measurement where the need for fractions and ratios emerged from the process of quantitative measurement as used by ancient Egyptians, Babylonians, Indians, Greeks and later Arabs. Fractional units are derived when the standard *unit of measure* is inadequate and the unit needs to be subdivided into smaller, equal sized divisions. These fractional units in combination with whole units provide an accurate means of measuring (Skemp, 1986).

During the process of measuring, algebraic relationships can also be established (Davydov, 1991). When comparing lengths A and B (see Figure 1), the relationships can expressed: $A < B$, $B > A$, $C = A + B$, $B = C – A$; $A = C – B$. If we wanted to measure B in relation to A which becomes our unit of measure, $B = A + A + A$ or $B = 3 \times A$. Many more algebraic relationships can be established. The relationship between the lengths of A and B also can be expressed as a ratio. The ratio of $A : B = 1 : 3$; conversely $A : B = \frac{1}{3} : 1$. Hence measurement is an everyday use of mathematics which provides a basis for developing understanding of whole number, fractions, ratios, algebra and mathematical reasoning.

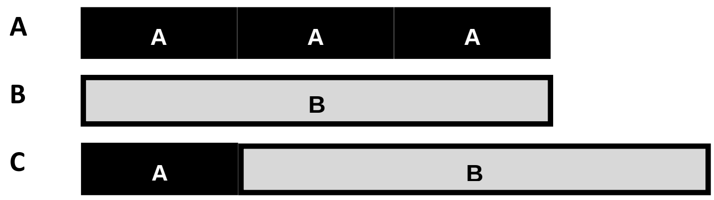

Figure 1. Comparing two lengths, A and B and establishing their relationship to C.

Mathematics in Yup'ik Everyday Context

When Yup'ik Elders are asked what Yup'ik word or concept best describes mathematics, they reply with *cuqete* (to measure) (Lipka, Mohatt, & Ciulistet., 1998). They considered mathematics as a tool necessary for their everyday survival. Their activities are firmly grounded in measurement where the body is central to the task. The body is a portable tool which enables any quantity to be reproduced at any place or time. Different *units of measure* derived from the body or local material are used depending on the activity. For instance the *yagneq*, *talinin*, *talyuaneq* and *ikugamek malruk* (see Figure 2) are used to describe the key measurements in the construction of a kayak to fit the maker (Lipka, Jones, Gilsdorf, Remick, & Rickard, 2010). These measures altogether form a proportional relationship between the person and the length and the width of the kayak.

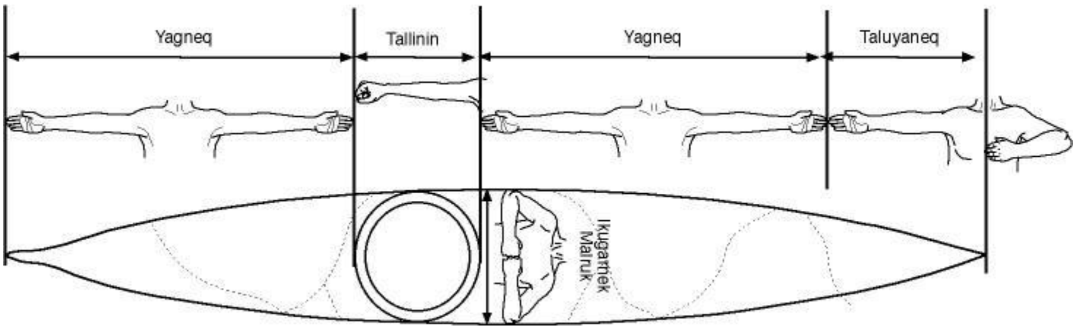

Figure 2. Designing a kayak to fit the user incorporates body proportional measuring.

Yup'ik seamstresses provide another example of symmetrical measuring. In ongoing research[2] with Yup'ik Elders, Mrs Nanalook, an elder, demonstrated and explained how she would measure both the second author (Lipka) and the third author (Andrew-Ihrke); if she was making a garment for each of them and she wanted to use the same unit of measure. She aligned Andrew-Ihrke and Lipka, Andrew-Ihrke standing in front of Lipka so that their *qukaq* [centre] were aligned. She measured the difference between them, using a body measure that she was very familiar with. She knew that Lipka's height minus Andrew-Ihrke's height equals the difference. She knew that, in this case, the difference would measure both of them. In fact, if d represents the difference in length between Lipka and Andrew-Ihrke then $4d$ equals Andrew-Ihrke's body length and $5d$ equals Lipka's body length. Lipka/Andrew-Ihrke is 5/4 while Andrew-Ihrke/Lipka is 4/5. Mrs Nanalook does not think of this numerical ratio but her measuring approach generates the ratio. Scaling and proportional measuring are central features of both the way Yup'ik people measure and MCC's approach to teaching from measuring. Thus, the introduction of mathematics through measurement has the potential to reconnect school mathematics to Yup'ik real world activities thus providing a learning context for AN students.

Davydov (1991), Barrett et al. (2012) and Dougherty and Venenciano (2007) have developed primary curriculum and examined students' development of their mathematical understandings from a measurement perspective. MCC's approach is unique. The elders provide a way of teaching mathematics that is quite similar to Davydov's approach yet simultaneously different. Elders have stated during meetings in 2013 and 2014 in Fairbanks that all projects and measuring begin from the centre. Mr. Jimmy, an elder from Mountain Village, crossed two fingers similar to the letter × which represents the "beginning of everything". Measuring symmetrically, halving, measuring as comparing, measuring as ratios and verification are the crucial processes within their everyday activities. Because of their conception, MCC begin our hypothetical learning/teaching sequence (see Table 1)

[2] National Science Foundation, Arctic Social Sciences Program, *The Potential Contribution of Indigenous Knowledge to the Teaching and Learning Mathematics* is a three-year grant working with five different culturally and linguistic groups (with the exception of Yup'ik in Alaska and Greenlandic Inuit) to document if symmetrical/proportional measuring is a basis for constructing a range of everyday artifacts. To date there is evidence across groups that symmetrical measuring and halving are key concepts and practices.

with this in the foremost of our minds. Importantly, this distinguishes MCC's work from the work of Davydov. The hypothetical learning/teaching sequence is informed by the work of Davydov but focuses on the knowledge and perspective of Yup'ik Elders, thus making it mathematically and culturally cohesive. To develop activities for the classroom based on the learning/teaching sequence, a number of activities were trialled at two of schools in rural interior Alaska. This paper describes two activities undertaken at a K-12 school with a focus on identifying the knowledge students exhibit and the opportunities for learning.

Table 1
Measuring as a Pathway for Connecting and Understanding Rational Number

Concept/Task	Knowledge
Measurement attributes	What is an attribute?
Identify attributes of objects that can be measured or compared with another object.	Objects have attributes other than length, mass, volume, temperature. Objects can be compared by the attribute.
Measurement	What is a unit? Unit is a specific measureable attribute and can be named.
What can we use to assist the measurement of attributes	Informal/formal units including standard measurement systems (e.g., metric; imperial)
Alternative measurement systems (e.g., body measures)	Quantity measured is related to the unit of measure.
Qualitative comparison (no numbers)	Need to identify object that you are measuring and what is the reference object.
Compare A and B.	More/less; taller/shorter; heavier/lighter… THAN requires comparison to a reference.
Qualitative comparison - Additive structure (no numbers)	Adding to, subtracting from to make both objects equal in relation to the attribute being compared.
Difference between A and B.	Algebraic reasoning $A + B = C$; $A \neq B$
How can we make them the same?	Addition and subtraction are inverse operations, e.g., $C – B = A$; $A + B = C$
Quantitative measurement using informal units	Chosen unit needs to be laid end to end, no gaps or overlaps.
Measure objects using an informal unit, A.	Measurement of object is the number of same-size length units.
Estimation	Quantitative measurements must be written with the appropriate units.

Table 1 (continued)
Measuring as a Pathway for Connecting and Understanding Rational Number

Concept/Task	Knowledge
Relationship between size of unit and number of units	Relationship between A and B, and predicting the length in B units.
If you have measured the table using unit A, how many unit B's do you need?	Smaller unit requires more of, to measure the same length and vice versa.
	Attribute of object being measured is invariant.
Predict and explain why.	Quantity changes depending on unit of measure.
Quantitative comparison - Multiplicative	Find a common unit of measure.
	Establish a ratio relationship between A and B.
Difference between A and B.	Multiplication and division – inverse relationship.
Scale	Zero
Creating a scale	Increments need to be equal.
	Sub-divide scale units to obtain fractional parts.
	Scale needs to be marked to indicate the unit of measure.

Measurement Activities

The School Setting

The Alaska Gateway School District (AGSD) is a small rural school district in interior Alaska. There are seven small schools in the district with a total student population of approximately 500 students (K-12); 58% of the student population are Alaska Native and the vast majority are Athabaskan, 38% are Caucasian and the remaining 4% are distributed across Asian, African-American, Hispanic, and American Indian. Mathematics performance of students in AGSD is at 57% proficiency level which is typically 7-10 percentage points lower than the state-wide average (during the past years). However, proficiency scores are even lower outside of the district's largest community. Two activities were conducted at the K-12 school within the district's largest community. Within the school there was one Grade 4 and one Grade 5 class with 14 and 12 students respectively.

Grade 4– Qualitative and Quantitative Comparison: Building a Foundation

The first activity was a foundational comparison task; make comparisons between two objects (K.MD.2) a kindergarten outcome (ABEED, 2012). But the activity moves onto quantitative comparison and additive strategies, in particular the inverse relationship of adding/subtracting. The activity was led by the second author and third author, a Yup'ik Elder. The activity was observed by the first author, another researcher and the classroom teacher. Author 2 and author 3 stand in front of the class.

Author 2 Jerry [J] asks, "Who is taller?"

James: You are 1 and ½ feet taller.

Ss: One's short, one's tall.

James: I think your [Jerry's] arms are longer. [Jerry and Dora stretch out their arms with their finger tips on their right hands aligned.]

J: What can you do to make us the same length?

Ss: Put your arms back [an action to shorten Jerry's arms]

James: Takeaway [talks about difference between Dora and Jerry's arm]

J: How about adding?

Valentine: Here's my theory.

J: What are you going to do to get us the same?

Valentine: Add at least 12 inch ruler to Dora [her armspan]

Dace: Shrink Jerry

Student Knowledge and Opportunities for Learning

Students were able to compare the height of two people. Although from an everyday perspective, we understand the meaning of "Jerry is taller" as we could see both people, the statement is mathematically incomplete as Jerry is taller "than who?" Dora. However, there will be instances when Jerry is not taller, depending on the other person. Correct mathematical language is fundamental to conveying information in a concise and unambiguous manner. The language used by students in their responses also provides opportunities to discuss related mathematical content. The comment "at least" enables the discussion of estimation and "one and a half" introduces the idea of fractional parts of the unit of measure and increasing accuracy in measuring.

The activity also demonstrated a focus on standard units with feet and inch used by the students. The process of measuring can be achieving by using any object with the required attribute under investigation. Linking measurement to the use of Yup'ik body measures as used by Mrs Nanalook and other cultural examples can be considered. What are the implications of different people having different lengths for a measure (e.g., foot) is a question for discussion and why standard units of measure have evolved.

Grade 4-5 – Relationship between size of unit and number of units

The second activity examined how measurements relate to the size of the unit chosen (2.MD.2), a grade 2 outcome (ABEED, 2012), but was more complicated as different units were used to measure different lengths. Prior to the activity, students traced their foot and took a measure of their height using adding machine tape. The activity explored students' height measured using their foot measure which is the length of their foot. The second author and the classroom teacher, Mrs T created a table on the board (see Table 2) to organise the data. A vertical line was drawn on the board to ensure students lined up the length of their foot measure so a visual representation [horizontal column graph] was simultaneously created. Author 2 traces his foot and shows the class his foot measure. Mrs T, with the help of students marked author 2's height with adding machine tape.

Author 2 Jerry [J]: I'm going to measure my length/height. What is my unit of measure? [The answer (his foot) is not immediately obvious.]

J: What else is a unit of measure? [Refers to the ruler]

J: This is MY foot. [holds up the tracing of his foot]

J then asks children to estimate his height with his foot. Students and Mrs T measure J's height with his foot measure. He enters his name and the length of the foot measure as a horizontal line within the table.

Students then measure their own height using their foot length. Students are told "Number of Units" column is in number of WHOLE units and they add their results to the table on the board. After the table is completed (a photo of the table was taken, however it is partially reproduced in Table 2). A number of observations were made by students after reviewing their table:

All children are around 6+ or 7+ feet tall.

Emily: "7+"; I'm taller than Jerry!

Mackenzie: Why is it that Kassi and I are the same height but she's 7 and I'm 6.

Table 2
Students' Height as Measured by their Foot Length

Name	Foot Measure	Number of Units	My Feet Tall
Author 2 [J]		6	6+
Emily		7	7+
Dace		6	$6\frac{1}{2}$
Mackenzie		6	6
Kassi		7	7

Student Knowledge and Opportunities for Learning

The activity engaged and actively involved students in both the physical process of measuring and actively thinking about different units that could be used to measure. As shown in activity 1, developing flexibility in thinking and using non-standard units of measure is needed when rulers and other standard measuring devices are unavailable. The activity created a mathematical dilemma for Emily. She was taller than the second author, who is 6 foot 4 inches (194 cm) because she was 7+ units tall whereas Author 2 was only 6+ units. To resolve part of Emily's confusion, students need to simultaneously co-ordinate the size of the unit and the number of units needed. Hence, the larger the unit, or longer the foot, the smaller the number needed to cover the length. The inverse is also true, the smaller the unit, the larger the number of units needed. This is true when using different units of measure to measure the same length. Emily's confusion was further complicated by the fact that not only does the unit of measure change in this activity, but so does the length that is being measured. Mackenzie makes a similar observation; she and Kassi are the same height, but their "My Feet Tall" is different.

The questions asked by the students provide opportunities for developing a deeper understanding of measurement and the need for fractions. Additionally, students were beginning to use ratios and measurement division in this activity. Their "foot" was the unit of measure (unknown length) and their height was the object to be measured (also an unknown length), thus $h \div f = \frac{h}{f} = x$. Students observed that the outcome of this division, dividend ÷ divisor results in a numerical quotient although both the divisor and the dividend were nonnumeric. Further, the height and foot relationship develops an understanding of ratios. The table and its construction also enabled the review of graphs. A graphing extension would be the modelling of the association between the two quantities "foot" vs height.

Conclusion

Although a limited set of data are presented from this preliminary work in developing curriculum materials from the perspective of measuring (symmetry, comparing, body proportionality, and verification), it is MCC's belief that this approach holds promise for teaching mathematics in a cohesive way for Alaskan Native students. Students were exposed to measuring using both additive and multiplicative structures in the activities presented; they began exploring the inverse relationship between the size and number of units. Their comments highlighted the understanding they possessed and the need to build flexibility in their thinking about measurement. The activities also created mathematical dilemmas for students and challenged their understanding. Opportunities to incorporate related mathematical content such as graphing and data was incorporated in the activities and other opportunities which arose from students' comments were identified. These foundational activities provide a springboard to explore Yup'ik activities (e.g., such as garment making) to ensure mathematics is contextually relevant for Alaskan Native students.

References

Alaska Board of Education and Early Development [ABEED]. (2012). Alaska mathematics standards (adopted June 2012). Retrieved 23 September, 2013, from http://education.alaska.gov/tls/assessment/2012AKStandards.html

Alaska Department of Education and Early Development. (n.d.). Spring 2011 standards based assessments. Retrieved 16 March, 2014, from http://www.eed.state.ak.us/tls/assessment/results/2011/statewide_sba.pdf

Barrett, J. E., Sarama, J., Clements, D. H., Cullen, C., McCool, J., Witkowski-Rumsey, C., & Klanderman, D. (2012). Evaluating and improving a learning trajectory for linear measurement in elementary grades 2 and 3: A longitudinal study. *Mathematical Thinking and Learning, 14*(1), 28-54.

Davydov, V. V. (1991). On the objective origin of the concept of fractions. *Focus on Learning Problems in Mathematics, 13*, 13-83.

Dougherty, B. J., & Venenciano, L. C. H. (2007). Measure up for understanding. *Teaching Children Mathematics, 13*(9), 452-456.

Juneau School District. (n.d.). Elementary math. Retrieved 15 March, 2014, from http://www.juneauschools.org/parents/elementary-math

Kisker, E. E., Lipka, J., Adams, B. L., Rickard, A., Andrew-Ihrke, D., Yanez, E. E., & Millard, A. (2012). The potential of a culturally based supplemental mathematics curriculum to improve the mathematics performance of Alaska native and other students. *Journal for Research in Mathematics Education, 43*(1), 75-113.

Lipka, J., & Adams, B. (2004). Culturally based math education as a way to improve Alaska native students' mathematics performance (Vol. Working Paper No. 20). Athens: Appalachian Center for Learning, Assessment, and Instruction in Mathematics.

Lipka, J., Jones, C., Gilsdorf, N., Remick, K., & Rickard, A. (2010). *Kayak design: Scientific method and statistical analysis*. Calgary, Alberta, Canada: Detselig Enterprises Ltd.

Lipka, J., Mohatt, G. V., & Ciulistet. (1998). *Transforming the culture of schools: Yup'ik Eskimo examples*. Mahwah, N.J.: Lawrence Erlbaum Associates.

Lipka, J., Yanez, E., Andrew-Ihrke, D., & Adam, S. (2009). A two-way process for developing effective culturally based math: Examples from math in a cultural context. In B. Greer, S. Mukhopadhyay, A. B. Powell & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 257-280). New York: Routledge.

National Center for Education Statistics. (2010). The nations' report card: Mathematics 2009 (NCES 2010-451). Washington, DC: Institute of Education Sciences, U.S. Department of Education.

Skemp, R. R. (1986). *The pyschology of learning mathematics* (2nd ed.). London: Penguin Books.

The National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the national mathematics advisory panel.* Washington, DC: U.S. Department of Education.